TTK4150 Nonlinear Control Systems
Exercise 5
Part 1

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Exercise 1
Consider the PID control law given in Proposition 2. Show that it is passive given the assumptions of the proposition.

Exercise 2
Consider the PID control law given in Proposition 3.

1. *Show that*
$$|h_{r2}(j\omega)| \leq \frac{K_p \beta}{\alpha} \ \forall \omega$$
given the assumptions of the proposition.

2. *Show that*
$$\mathrm{Re}\left[h_{r2}(j\omega)\right] \geq K_p \ \forall \omega$$
given the assumptions of the proposition.

3. *Show that it is passive given the assumptions of the proposition.*

4. *Show that it is input strictly passive given the assumptions of the proposition.*

5. *Show that it is output strictly passive given the assumptions of the proposition.*

6. *Show that the system is zero-state observable given the assumptions of the proposition.*

Exercise 3
Comment on stability of the closed loop for each of the following cases, when closed with h_{r1} and h_{r2}

1. *The plant is passive*
2. *The plant is input strictly passive*
3. *The plant is output strictly passive*
4. *The plant is output strictly passive and zero-state observable*
5. *The plant is strictly passive*

Exercise 4 (Exercise 6.11 in Khalil)
Hint: Use $V(\omega) = \frac{1}{2}J_1\omega_1^2 + \frac{1}{2}J_2\omega_2^2 + \frac{1}{2}J_3\omega_3^2$

Exercise 5 (Exercise 6.14 in Khalil)
Hint: Use $V_1(x_1, x_2) = \frac{1}{2}(x_1^2 + x_2^2)$ *and* $V_2(x_3) = \int_0^{x_3} h_2(z)\,dz$

Exercise 6 (Exercise 6.15 in Khalil)
Hint: Use $V_1(x_1, x_2) = \frac{1}{4}x_1^4 + \frac{1}{2}x_2^2$ *and* $V_2(x_3) = \frac{1}{4}x_3^4$

A Passive systems

A.1 Linear systems

Theorem 1
Given a linear time-invariant system

$$y(s) = h(s)\, u(s)$$

with a rational transfer function $h(s)$. *Assume that all the poles of* $h(s)$ *have real parts less than zero. Then the following assertions hold:*

- *The system is passive* $\Leftrightarrow \operatorname{Re}[h(j\omega)] \geq 0\ \forall\omega$
- *The system is input strictly passive* $\Leftrightarrow \operatorname{Re}[h(j\omega)] \geq \delta \geq 0\ \forall\omega$
- *The system is output strictly passive* $\Leftrightarrow \operatorname{Re}[h(j\omega)] \geq \varepsilon\, |h(j\omega)|^2\ \ \forall\omega$

Proposition 1
Consider a rational transfer function

$$h(s) = \frac{(s+z_1)(s+z_2)\cdots}{s(s+p_1)(s+p_2)\cdots}$$

where $\operatorname{Re}[z_i] > 0$ *and* $\operatorname{Re}[p_i] > 0$. *Then the following assertions holds:*

- *The system is passive* $\Leftrightarrow \operatorname{Re}[h(j\omega)] \geq 0\ \forall\omega$

A.2 Passivity of the PID controllers

Proposition 2
Assume that $K_p > 0$, $0 \leq T_d < T_i$ *and* $0 \leq \alpha \leq 1$. *Then the PID controller*

$$h_{r1}(s) = K_p \frac{(1+T_i s)(1+T_d s)}{T_i s (1+\alpha T_d s)}$$

is passive.

Proposition 3
Assume that $K_p > 0$, $0 \leq T_d < T_i$, $1 \leq \beta < \infty$ *and* $0 < \alpha \leq 1$. *Then the PID controller*

$$h_{r2}(s) = K_p \beta \frac{(1+T_i s)(1+T_d s)}{(1+\beta T_i s)(1+\alpha T_d s)}$$

is passive. Further, it satisfies the following:

- $|h_{r2}(j\omega)| \leq \frac{K_p \beta}{\alpha}\ \forall\omega$
- $\operatorname{Re}[h_{r2}(j\omega)] \geq K_p\ \forall\omega$