TTK4150 Nonlinear Control Systems
Exercise 4

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Fall 2003

Exercise 1
Let

$$
\begin{aligned}
V_1(x_1, x_2 t) &= x_1^2 + \left(1 + e^t\right) x_2^2 \\
V_2(x_1, x_2 t) &= \frac{x_1^2 + x_2^2}{1 + t} \\
V_3(x_1, x_2 t) &= \left(1 + \cos^4 t\right) \left(x_1^2 + x_2^2\right)
\end{aligned}
$$

For each of the functions $V_i(x_1, x_2 t)$, $i \in \{1, 2, 3\}$ *investigate the properties of positive definite and decresent.*

Exercise 2
Consider the system

$$
\begin{aligned}
\dot{x}_1 &= -x_1 - g(t) x_2 \\
\dot{x}_2 &= x_1 - x_2
\end{aligned}
$$

where the function $g(t)$ *is continuous differentiable and satisfies*

$$0 \leq g(t) \leq k \text{ and } \dot{g}(t) \leq g(t) \;\; \forall t \geq 0$$

and k *is a positive constant. Investigate the stability properties of the origin by using the function*

$$V(t, x) = x_1^2 + (1 + g(t)) x_2^2$$

Exercise 3
Consider the system

$$
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -x_1 - c(t) x_2
\end{aligned}
$$

where the function $c(t)$ *is continuous differentiable and satisfies*

$$k_1 \leq c(t) \leq k_2 \text{ and } |\dot{c}(t)| \leq k_3 \; \forall t \geq 0$$

and k_i *are constants and* $k_1 > 0$. *Use the Lyapunov function candidate*

$$V(x) = \frac{1}{2}\left(x_1^2 + x_2^2\right)$$

to show that the origin is uniformly stable and that $x_2 \to 0$ *as* $t \to \infty$.

Exercise 4 (Exercise 4.55 in Khalil)

Exercise 5 (Exercise 5.1 in Khalil)

Exercise 6 (Exercise 5.2 in Khalil)

Exercise 7 (Exercise 5.21 in Khalil)

Exercise 8 (Exercise 6.1 in Khalil)

Exercise 9 (Exercise 6.4 in Khalil)

Exercise 10 (Exercise 6.6 in Khalil)

Exercise 11 (Exercise 6.10 in Khalil)