TTK4150 Nonlinear Control Systems
Exercise 1

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Exercise 1 (Exercise 3.2 in Khalil)

Exercise 2 (Exercise 3.16 in Khalil)

Exercise 3 (Exercise 2.2 in Khalil)

Exercise 4
For the systems 1 and 2 in Exercise 3, construct the phase portrait and discuss the qualitative behavior of the system.

Exercise 5 (Exercise 2.21 in Khalil)

Exercise 6 (Exercise 2.24 in Khalil)

Exercise 7
A synchronous generator (for instance used on a water or gas power plant) connected to a infinite bus (no dynamics in the net) can be described by the equations

$$\begin{aligned} M\ddot{\delta} &= P - D\dot{\delta} - \eta_1 E_q \sin\delta \\ \tau \dot{E}_q &= -\eta_2 E_q + \eta_3 \cos\delta + E_{FD} \end{aligned}$$

where δ a angle related to the rotor (a simplification states $\dot{\delta} = \omega_{rotor} - \omega_{net}$), E_q is voltage, E_{FD} is the field current voltage (manipulated input), P is mechanical effect (for instance given by the water running through the turbine),

D is a damping coefficient, M a energy constant, τ is a time constant and η_i are various constants. The control objective is to maintain a constant frequency ($\dot{\delta} = 0$) at a desires voltage.

1. *Find the state space model using P and E_{FD} as inputs.*
2. *Use*

$$\begin{aligned} P &= 0.815 \\ E_{FD} &= 1.22 \\ M &= 0.0147 \\ \frac{D}{M} &= 4 \\ \eta_1 &= 2 \\ \eta_2 &= 2.7 \\ \eta_3 &= 1.7 \end{aligned}$$

   and calculate the equilibrium points. The parameters given represents normalized parameters where it is desirable to have $E_q \approx 1$ and a small angle δ. Comment on the equilibrium points found.

Exercise 8
An auto pilot for a large tanker is shown together with a detailed block diagram in Figure 1. Symbols used in the figure is explained in Table 1.

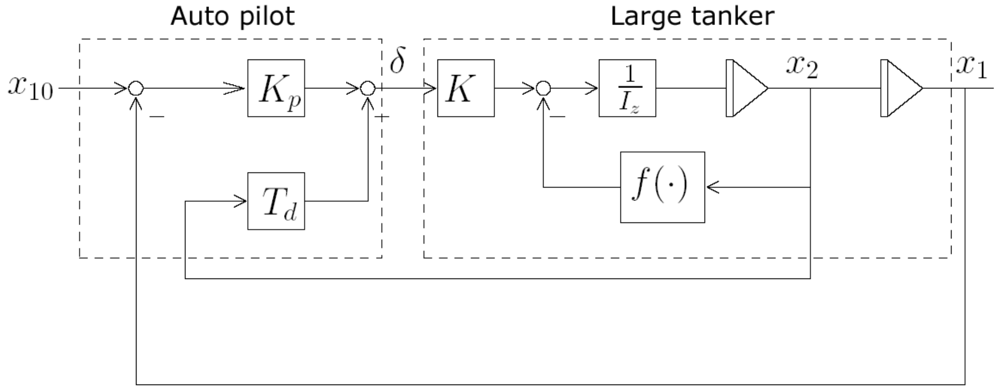

Figure 1:

1. *Find the state space model of the system in Figure 1.*

x_{10}	course reference
x_1	course angle
x_2	angular velocity
$f(x_2)$	hydrodynamic damping
I_z	moment of inertia
K	rudder constant
K_p	controller gain ($K_p > 0$)
T_d	controller derivative time constant ($T_d > 0$)

Table 1: Symbols used in Figure 1

2. *In the remaining of this exercise we will use*

$$\begin{aligned} K &= I_z = 1 \\ f(x_2) &= -x_2 + x_2\,|x_2| \end{aligned}$$

   Find the equilibrium points of the system.

3. *Derive the type of equilibrium points expressed in terms of K_p and T_d, and show the result in a diagram in the (K_p, T_d)-plane.*

4. *What type of equilibrium point correspond to the controller values $K_p = T_d = 4$. Draw the trajectories of the following initial conditions in the phase plane when the system is exposed to a step from $x_1^* = 0$ to $x_1^* = 2$.*

   (a) $x_1 = x_2 = 0$

   (b) $x_1 = x_2 = 2$