ESI, October 23, 2002
Institutt for Teknisk Kybernetikk
NTNU, Trondheim

SIE 3015 - Linear Systems

Voluntary Assignment 5
Transfer functions, state-space realizations and state estimation

Handed out:	October 24th
Tutorial:	October 31st and November 7th
Hand in by:	November 8th

It is recommended that you try to solve the problems by hand, but feel free to use MATLAB to verify your results. These MATLAB functions are particularly relevant: `ss2tf`, `tf2ss`, `minreal`, `lyap`.

Exercise 1. Transfer functions and state-space realization
Given the following system

$$Y(s) = \frac{1}{s-3}\left(U(s) + V(s)\right) \tag{1}$$

with disturbance $V(s)$ and controller given by

$$U(s) = \frac{s-3}{s}(R(s) - Y(s)) \tag{2}$$

(a) Draw a block diagram and find the transfer function from $R(s)$ to $Y(s)$ and from $V(s)$ to $Y(s)$.

(b) Find the transfer function from $R(s)$ to $U(s)$.

(c) Assume $V(s) = 0$. Find a minimal realization for the system with the given controller.

(d) Using the results from (a)-(c), what can you say regarding stability of this control system?

(e) Simulate the system using SIMULINK.

Exercise 2. State estimation
Given a first order system ($n = 1$) with a state estimator and feedback from estimated states:

$$\dot{x} = ax + bu \tag{3}$$

$$\dot{\hat{x}} = a\hat{x} + bu + l(y - c\hat{x}) \tag{4}$$

$$u = -k\hat{x} + r \tag{5}$$

$$y = cx \tag{6}$$

(a) Show that

$$\frac{Y(s)}{R(s)} = c(s - a + bk)^{-1}b \tag{7}$$

(b) Using the result from (a), what is the practical significance of the state estimator with regards to the performance of the control system?

Remark: The assumption $n = 1$ is only to simplify the calculations. The results can be derived also for $n > 1$.