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Institutt for Teknisk Kybernetikk
NTNU, Trondheim

SIE 3015 - Linear Systems

Voluntary Assignment 4
State feedback and state estimation

Handed out:	October 10th
Tutorial:	October 17th and 24th
Hand in by:	October 25th

It is recommended that you try to solve the problems by hand, but feel free to use MATLAB to verify your results. These MATLAB functions are particularly relevant: `ctrb`, `obsv`, `place`, `eig` and `tf2ss`.

Exercise 1. Controlling the motion of a ship
In this exercise you will control the position of a ship. To make things easier, we assume that the ship's movement is limited to one degree of freedom (1-DOF). State estimation is necessary since it is important to be able to determine the ship's velocity and the external forces due to currents and wind.

A simplified mathematical model of the ship's position in one DOF can be written as:

$$\begin{aligned} x_1(k+1) &= x_1(k) + x_2(k) \\ x_2(k+1) &= x_2(k) + x_3(k) + u(k) \\ x_3(k+1) &= x_3(k) + w(k) \\ y(k) &= x_1(k) + v(k) \end{aligned}$$

The following normalized quantities have been used:

$x_1(k)$	–	The ship's position
$x_2(k)$	–	The ship's velocity
$x_3(k)$	–	Forces from currents and wind (disturbances)
$u(k)$	–	Propeller forces
$w(k)$	–	Change in the disturbance from time k to $k+1$
$y(k)$	–	Measurement of the ship's position (using a GPS)
$v(k)$	–	Deviation of the position measurement

(a) Are all the system's states observable?

(b) Make a state estimator (observer) with poles at $p_1 = 0.4$, $p_2 = 0.5$ and $p_3 = 0.6$.

(c) Show that the system is not controllable. Which states are uncontrollable?
Try to give a physical explanation for this.

(d) Design a controller for the ship's position which attempts to cancel the disturbance using the estimated disturbance:

$$u(k) = -\hat{x}_3(k) + \overline{u}(k)$$

where $\overline{u}(k)$ is a feedback control signal from the estimated states $\hat{x}_1(k)$ and $\hat{x}_2(k)$:

$$\overline{u}(k) = K_1\hat{x}_1(k) + K_2\hat{x}_2(k)$$

Select the poles of the controllable part of the system to be in $p = 0.7 \pm 0.2i$.

(e) Is the resulting control system with feedback from the estimated states stable? Determine the resulting poles.

(f) Simulate the system with state estimator and controller (use SIMULINK).
Assume the measurement noise v is a Gaussian distributed random signal (white noise) (use the SIMULINK block "sources $\rightarrow$ random number" with variance 0.001, mean value 0.0 and sampling time 1), and the disturbance w is a sinusoidal signal with frequency 1.2 rad/s and amplitude 0.05.

(g) Does the controller have an integrating effect?

Exercise 2. Realizations and canonical forms
(a) Determine a state-space representation with only 2 states for the system given by the following transfer function:

$$\frac{Y(z)}{U(z)} = \frac{1.2 + 3.4z^{-2}}{1 + 0.8z^{-1} + z^{-2}}$$

(b) Transform the system you found on state-space form in (a) to diagonal canonical form.

(c) Assume $\frac{Y(z)}{U(z)}$ is the transfer function of a controller. Write a pseudo-program for the controller based on the state-space realization from (a).

Exercise 3. Realizations
Find a state-space representation for the system given by the following transfer function matrix:

$$G(s) = \left[\frac{(1+2s)(1-s)}{(1+s)(1+4s)}, \; \frac{1}{s(1+s)}\right]$$

The system has two inputs and one output.

Exercise 4. Controllability and linear algebra
(a) Show that the controllability matrix

$$C_k = \left(B, AB, A^2B, ..., A^{k-1}B\right)$$

has same rank for all $k \geq n$.

(b) Chen 3.5

(c) Chen 3.13

(d) Chen 3.14

(e) Chen 3.21

Answers to exercises

Exercise 1
(a) Yes
(b) $l = [1.5\ 0.74\ 0.12]^T$
(c) x_3 is uncontrollable
(d) $k = [0.13\ 0.6\ 1]$
(e) Stable
(f) Yes

Exercise 2
(a)

$$\begin{aligned} \mathbf{x}(k+1) &= \begin{bmatrix} -0.8 & -1 \\ 1 & 0 \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \mathbf{u}(k) \\ \mathbf{y}(k) &= \begin{bmatrix} -0.96 & 2.2 \end{bmatrix} \mathbf{x}(k) + 1.2\mathbf{u}(k) \end{aligned}$$

(b)

$$\begin{aligned} \mathbf{x}(k+1) &= \begin{bmatrix} -0.4 & 0.91652 \\ -0.91652 & -0.4 \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} -1.543 \\ 0 \end{bmatrix} \mathbf{u}(k) \\ \mathbf{y}(k) &= \begin{bmatrix} 0.62215 & 1.8272 \end{bmatrix} \mathbf{x}(k) + 1.2\mathbf{u}(k) \end{aligned}$$

Exercise 3

$$\begin{aligned} \dot{\mathbf{x}} &= \begin{bmatrix} -1.25 & 0 & -0.25 & 0 & 0 & 0 \\ 0 & -1.25 & 0 & -0.25 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \mathbf{u} \\ \mathbf{y} &= \begin{bmatrix} 0.875 & 0 & 0.375 & 1 & 0 & 0.25 \end{bmatrix} \mathbf{x} + \begin{bmatrix} -0.5 & 0 \end{bmatrix} \mathbf{u} \end{aligned}$$