ESI, October 3, 2002
Institutt for Teknisk Kybernetikk
NTNU, Trondheim

SIE 3015 - Linear Systems

Voluntary Assignment 3
w-transform, solving state equations, canonical forms

Handed out:	September 26th
Tutorial:	October 3rd and 10th
Hand in by:	October 11th at 12.00 in dedicated locker, building B

It is recommended that you try to solve the problems by hand, but feel free to use MATLAB to verify your results. These MATLAB functions are particularly relevant: `ss`, `tf`, `step`, `canon`, `ss2ss`, `ltiview`, `bode` and `c2d`.

Exercise 1. w-transform
Given the following system:

$$G(s) = \frac{5}{1+2s}$$

Using the w-transform, design a digital PI-controller which satisfies the following requirements: Phase margin of at least 40 degrees and gain margin of at least 6 dB. Assume the sampling interval is $T = 0.8$s and find the controller algorithm for the digital PI-controller that yields the highest possible cross-over frequency while satisfying the given requirements.
Determine the resulting cross-over frequency.

Exercise 2. Solving linear state equations
Given the following nonlinear system with 2 controlled inputs and 2 states (this is a simple model of a chemical reactor where x_1 is the compound, x_2 is temperature, u_1 is feed and u_2 is input heat.):

$$\dot{x}_1 = u_1(2 - x_1) + x_1 e^{x_2}$$
$$\dot{x}_2 = u_2(4 - x_2) - 2x_1 e^{x_2}$$

(a) Using linearization, find a continuous state space model which is valid close to the point of equilibrium given by $x_1^* = x_2^* = 1$. The model can be written as:

$$\frac{d}{dt}\Delta x = A\Delta x + B\Delta u \tag{1}$$

where A and B are 2 x 2-matrices. The variables $\Delta x = (x_1 - x_1^*, x_2 - x_2^*)^T$ and $\Delta u = (u_1 - u_1^*, u_2 - u_2^*)^T$ define the deviation from the point of equilibrium.

Find u_1^* and u_2^*.

(b) Assume that $\Delta u(t) = (0,0)^T$ for all t and $\Delta x(0) = (0.5, 0.3)^T$. Solve the continuous linear state equation and determine $x(t = 2)$.

(c) Perform an exact discretization of the continuous linear system on state space form using a sampling interval $T = 0.5$.

(d) Solve the discrete state equation for $\Delta x(k = 4)$ using the came conditions as in part (b) above.

(e) Compare the discrete and the continuous solution using simulations.

(f) Compare your results with the nonlinear system using simulations.

Exercise 3. Canonical forms
Given the system $\dot{x} = Ax + Bu$ where

$$A = \begin{pmatrix} -2 & 1 \\ 3 & 0 \end{pmatrix}, B = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

(a) Find an equivalent diagonal state space representation for the system. Draw a block diagram.

(b) Find an equivalent state space representation for the system using observable canonical form. Draw a block diagram.