ESI, September 9, 2002
Institutt for Teknisk Kybernetikk
NTNU, Trondheim

SIE 3015 - Linear Systems

Voluntary Assignment 2
Discrete systems and controller-design

Handed out:	September 12th
Tutorial:	September 19th and 26th
Hand in by:	September 27th at 12.00 in dedicated locker, building B

It is recommended that you solve the problems by hand, but feel free to use MATLAB to verify your results. These MATLAB functions are particularly relevant: `tf`, `ltiview` and `c2d`.

Exercise 1. Difference equations
Solve the difference equation

$$2y(k) - y(k-1) - y(k-2) = u(k-1) + 2u(k-2)$$

when $y(k) = 0$ for $k < 0$ and

$$u(k) = \begin{cases} 1, & k \geq 0 \\ 0, & k < 0 \end{cases}$$

Exercise 2. Sampling
Assume that the following signal is sampled with a sampling period T:

$$x(t) = 2\sin(3t) + \sin(8t)$$

(a) What value must T have in order to be able to exactly reconstruct the original signal from the sampled data $x(kT)$?

(b) Assume $T = 0.5$s. Determine the continuous signal that results from a reconstruction from the sampled data using an ideal lowpass filter with break frequency $\omega_s/2$. Discuss the result.

(c) Describe means that are helpful for avoiding folding when sampling.

Exercise 3. Poles and zeros in discrete systems
(a) Given the system

$$G_1(s) = \frac{1+0.5s}{s(1+2s)}$$

Find the corresponding discrete system (with a ZOH). Draw a diagram showing the placement of the poles and zeros in both the s- and the z-plane. Determine which poles/zeros in the z-plane correspond to the poles/zeros in the s-plane. Determine the step response for the continuous and for the discrete system

(b) Repeat the problems in (a) with the following system:

$$G_2(s) = \frac{s}{1 + 0.4s + 4s^2}$$

Exercise 4. Digital controller-design using pole placement
Given the system

$$\frac{Y(z)}{U(z)} = \frac{z^{-1}}{(1 - 0.8z^{-1})(1 - 0.6z^{-1})}$$

Assume a discrete controller given by:

$$U(z) = (k_1 + k_2 z^{-1})(R(z) - Y(z))$$

where k_1 and k_2 are (as of yet undefined) control parameters and $r(k)$ is a reference signal.

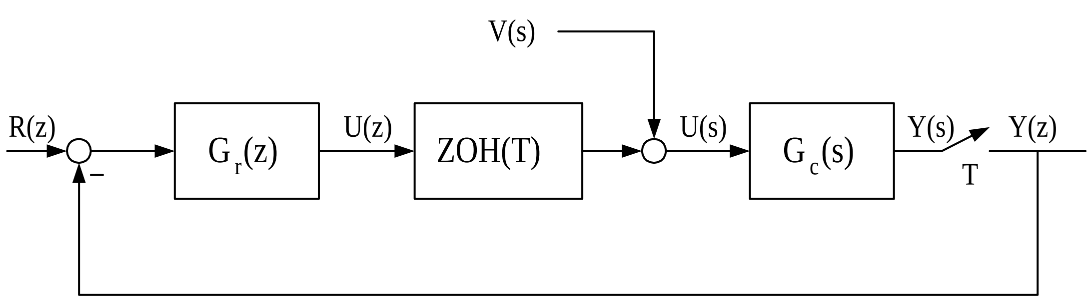
(a) Determine the transfer function $Y(z)/R(z)$.

(b) Assume that we want the closed-loop system $Y(z)/R(z)$ to have poles in $p_1 = 0.6$ and $p_2 = 0.5$. Determine the values of k_1 and k_2 to achieve this.

(c) Study the step response and the frequency response for the controller you found in (b) when $T = 0.1$. What is the relation between these responses and the chosen poles p_1 and p_2? How do zeros affect the responses?

Exercise 5. Transfer functions and digital controllers
It will probably be a good idea to use MATLAB to solve some of the problems. MATLAB can also be used to verify your answers (but remember that you will not be allowed to use MATLAB for the exam). These MATLAB functions are particularly helpful: `tf`, `c2d`, `feedback`, `series`, `bode`, `step`, `ltiview`.

Given the following system containing a sample-and-hold circuit:



The sampling period is T and

$$G_c(s) = \frac{1}{s(s+2)}$$

(a) Determine the transfer function $G_d(z) = Y(z)/U(z)$.

(b) Determine the impulse response $g_d(k)$.

(c) Compare the frequency response $G_d(e^{j\omega})$ of the discrete system with the frequency response $G_c(j\omega)$ of the continuous system as T varies. Use MATLAB if you want.

(d) Design a continuous PD controller

$$G_r(s) = K_p \frac{1 + T_d s}{1 + \alpha T_d s}$$

where $\alpha = 0.2$, so that the complete open-loop system (controller + process) satisfies $\omega_c = 5 rad/s$ and has a phasemargin of at least 30^{o}. Feel free to use MATLAB.
Note: The requirement on the crossing frequency ω_c may not be an optimal utilization of the controller. Can you see why?

(e) Discretize the PD-controller from above, write out the controller algorithm and investigate performance and stability as T varies. It may be a good idea to use frequency response tools in MATLAB.

(f) Use the discrete final-value theorem to determine the steady-state error that results from having a unit-step on the disturbance $v(t)$.

(g) Determine the response to a unit-step on the disturbance $v(t)$ and on the reference $r(k)$. Use MATLAB if you want.