Computer simulations of effective lattice theories for strongly correlated systems

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List of papers

Paper I:
S. Kragset, A. Sudbø, and F. S. Nogueira,
Metal-insulator transition in two- and three-dimensional logarithmic plasmas,
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Paper I:
*Metal-insulator transition in two- and three-dimensional logarithmic plasmas*

Paper II:
K. Børkje, S. Kragset, and A. Sudbø,
*Instanton correlators and phase transitions in two- and three-dimensional logarithmic plasmas,*
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Paper I:
Metal-insulator transition in two- and three-dimensional logarithmic plasmas

Paper II:
Instanton correlators and phase transitions in two- and three-dimensional logarithmic plasmas

Paper III:
S. Kragset, E. Smørgrav, J. Hove, F. S. Nogueira and A. Sudbø,
First order phase transition in a gauge theory of $S = 1/2$ quantum antiferromagnets,
cond-mat/0609336 (accepted for publication in Physical Review Letters)
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Paper IV:
S. Kragset, E. Babaev, and A. Sudbø,
Thermal fluctuations of vortex matter in trapped Bose–Einstein condensates,
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Strongly correlated systems

Resistance

Temperature $T$
Strongly correlated systems

- 1911: Superconductivity discovered

- Theory explaining the mechanism in 1957

- 1986: Discovery of high-temperature superconductors — theory still missing

$T_C$  

Temperature $T$

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Resistence

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  — theory still missing
High-temperature superconductors

- Oxygen
- Copper
- Lanthanum

Complex structures:
\( \text{La}_2\text{CuO}_4 \)

Temperature

Isolator
High-temperature superconductors

- Complex structures: $\text{La}_2\text{CuO}_4$
- Doping: $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$

![Diagram showing the structure of high-temperature superconductors with Oxygen, Copper, and Lanthanum atoms, along with a phase diagram showing the transition from Isolator to Supercond. as a function of Doping $x$.](image-url)
High-temperature superconductors

- Almost independent 2D planes
- Construct effective lattice models
Effective lattice models

— Superconductivity is a macroscopic property
Effective lattice models

Example: Particle description of light

- Superconductivity is a macroscopic property
- Simplify microscopic model
Effective lattice models

Example: Particle description of light

- Superconductivity is a macroscopic property
- Simplify microscopic model
- Put everything on a lattice
Computer simulations

— Model systems are not physical
Computer simulations

— Model systems are not physical
— Models are simplified, but not that simple

\[ \langle O \rangle = \frac{1}{Z} \sum_{\{\psi\}} O_\psi e^{-H_\psi / T} \]
Computer simulations

— Model systems are not physical
— Models are simplified, but not that simple

\[
\langle O \rangle = \frac{1}{Z} \sum_{\{\psi\}} O_{\psi} e^{-H_{\psi}/T}
\]

— Use computers for experiments
Computer simulations
— Monte Carlo

\[ \langle O \rangle = \frac{1}{Z} \sum_{\{\psi\}} O_\psi e^{-H_\psi / T} \] — Statistical methods
Computer simulations
— Monte Carlo

\[ \langle O \rangle = \frac{1}{Z} \sum_{\{\psi\}} O_\psi e^{-H_\psi / T} \]

— Statistical methods
— Temperature enhances fluctuations
**Computer simulations — Monte Carlo**

\[
\langle O \rangle = \frac{1}{Z} \sum_{\{\psi\}} O_{\psi} e^{-H_{\psi}/T}
\]

— Statistical methods
— Temperature enhances fluctuations
— Monte Carlo simulations imitate nature’s randomness
Paper I and II:
A logarithmically interacting plasma
Paper I and II: A logarithmically interacting plasma
Dipole moment

— Polarizability proportional to $\langle s^2 \rangle$
Dipole moment

— Polarizability proportional to $\langle s^2 \rangle$
— Measure $\langle s^2 \rangle$ as a function of temperature
Dipole moment

- Polarizability proportional to $\langle s^2 \rangle$
- Measure $\langle s^2 \rangle$ as a function of temperature
- System size upper limit
Inverse dielectric constant

\[ \epsilon^{-1}(k) \]

\( T \)

\( L = 10 \)

\( L = 30 \)

\( L = 50 \)

\( L = 70 \)

\( L = 100 \)
Inverse dielectric constant

\[ \epsilon^{-1}(k) \]

\[ T = 1.2 \text{ to } 1.9 \]

\[ L = 10, 30, 50, 70, 100 \]

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S. Kragset, Effective lattice theories
Paper IV: Thermal fluctuations of vortex matter in trapped Bose–Einstein condensates

— Some atomic gases condense into superfluids at extremely low temperatures

J. Dalibard
Some atomic gases condense into **superfluids** at extremely low temperatures.

Does not respond to rotation.

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J. Dalibard
Paper IV: Thermal fluctuations of vortex matter in trapped Bose–Einstein condensates

— Some atomic gases condense into superfluids at extremely low temperatures
— Does not respond to rotation
— Not until a certain rotation rate

J. Dalibard
Paper IV:
Thermal fluctuations of vortex matter in trapped Bose–Einstein condensates

— Some atomic gases condense into superfluids at extremely low temperatures
— Does not respond to rotation
— Not until a certain rotation rate
— Faster rotation

J. Dalibard
Vortices

— Dynamics of neutron stars
— Type-II superconductors
— Rotating Bose condensates and other superfluids
Vortex lattices

— Difficult to control temperature in experiments
— What are the temperature effects?
— Convenient to use Monte Carlo simulations
Modelling vortex systems

— Simple model of phase fluctuations
Simulated trapped vortex lattices

— Condensate density depletes in outer regions with increasing temperature
— High stability in the lattices
Conclusions

With Monte Carlo simulations I have studied:

— Phase transitions in effective lattice models for strongly correlated systems
— Thermal fluctuations in trapped Bose–Einstein condensates

The results have been published in four research articles